

Lógica I (FIL 120)

Universidade Federal de Ouro Preto

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Exercícios parcialmente resolvidos

Lógica de predicados com identidade

Páginas 182–184

Lógica: Um Curso Introductório, de W. H. Newton-Smith (Gradiva, 1998)

1.

- a) $\forall x (Zx \rightarrow Cx)$
- b) $\forall x (Zx \rightarrow \neg Cx)$
- c) $\forall x (Vx \rightarrow Cx) \quad / \quad \exists x (Vx \wedge \forall y (Vy \rightarrow y = x) \wedge Cx)$
- d) $\exists x (Cx \wedge \forall y (Cy \rightarrow y = x) \wedge Vx)$
- e) $\forall x Anx$
- f) $\exists x \forall y (Ixy \wedge \forall z (Azy \rightarrow z = x) \wedge x = n)$
- g) $\forall x \forall y (Cxy \rightarrow Txy)$
- h) $\forall x (Cx \rightarrow Lx) \quad / \quad \exists x (Cx \wedge \forall y (Cy \rightarrow y = x) \wedge Lx)$
- i) $\exists x (Pxn \wedge \forall y (Pyn \rightarrow y = x) \wedge \exists z (Mzm \wedge \forall y (Mym \rightarrow y = z) \wedge Fxz)$
- j) $\forall x ((Tx \wedge \exists y (Px \wedge \forall z (Pz \rightarrow z = y))) \rightarrow Dxy)$
- k) $\exists x (Px \wedge \forall y (Py \rightarrow y = x) \wedge Ex)$
- l) $\exists x \forall y ((Cx \wedge Cy) \rightarrow Mxy) \wedge \forall z ((Cz \wedge Mzy) \rightarrow z = x) \wedge \exists w \forall v (Awv \wedge \forall z (Azv \rightarrow z = w) \wedge x = w)$
- m) $\exists x (Rx \wedge \forall y (Ry \rightarrow y = x) \wedge Px)$
- n) $\neg \exists x (x = n)$
- o) $\neg \exists x Mxn$

2.

- a) $Cnm, m = o \vdash Cno$

Prem 1. Cnm

Prem 2. $m = o$

1, 2 3. Cno 1, 2, $E=$

b) $\neg\exists x (Sxn), m = n \vdash \neg\exists x (Sxm)$

Prem 1. $\neg\exists x (Sxn)$
 Prem 2. $m = n$
 1, 2 3. $\neg\exists x (Sxm)$ 1, 2, $E=$

c) $\forall x (\neg Gxx \rightarrow Gnx) \vdash Gnn$

Prem	1.	$\forall x (\neg Gxx \rightarrow Gnx)$	
Sup	2.	$\neg Gnn$	
1	3.	$\neg Gnn \rightarrow Gnn$	1, $E\forall$
1, 2	4.	Gnn	2, 3, $E\rightarrow$
1, 2	5.	$Gnn \wedge \neg Gnn$	2, 4
1	6.	Gnn	2-5, $I\neg$

d) $\forall x (Rx \rightarrow Bnx), Rm, o = m \vdash Bno$

Prem 1. $\forall x (Rx \rightarrow Bnx)$
 Prem 2. Rm
 Prem 3. $o = m$
 1 4. $Rm \rightarrow Bnm$ 1, $E\forall$
 1, 2 5. Bnm 2, 4, $E\rightarrow$
 1, 2, 3 6. Bno 3, 5, $E=$

e) $\forall x (Vx \rightarrow Rnx), n = m \vdash \forall x (Vx \rightarrow Rmx)$

Prem 1. $\forall x (Vx \rightarrow Rnx)$
 Prem 2. $n = m$
 1, 2 3. $\forall x (Vx \rightarrow Rmx)$ 1, 2, $E=$

f) $\exists x (Rx \wedge \forall y (Ry \rightarrow y = x) \wedge Cx \wedge n = x) \vdash Cn$

Prem	1.	$\exists x (Rx \wedge \forall y (Ry \rightarrow y = x) \wedge Cx \wedge n = x)$	
Sup	2.	$Ra \wedge \forall y (Ry \rightarrow y = a) \wedge Ca \wedge n = a$	
2	3.	Ca	2, E \wedge
2	4.	$n = a$	2, E \wedge
2	5.	Cn	3, 4, E= $$
1	6.	Cn	1, 2-5, E \exists

g) $\exists x (Rx \wedge \forall y (Ry \rightarrow y = x) \wedge Fx) \vdash \exists x Rx$

Prem	1.	$\exists x (Rx \wedge \forall y (Ry \rightarrow y = x) \wedge Fx)$	
Sup	2.	$Ra \wedge \forall y (Ry \rightarrow y = a) \wedge Fa$	
2	3.	Ra	2, E \wedge
2	4.	$\exists x Rx$	3, I \exists
1	5.	$\exists x Rx$	1, 2-4, E \exists

h) $\exists x \exists w \forall y (Bxy \wedge Tx \wedge \forall z ((Bzy \wedge Tz) \rightarrow z = x) \wedge Px \wedge Bwy \wedge Mw \wedge \forall z ((Bzy \wedge Mz) \rightarrow z = w) \wedge w = x) \vdash \exists x \forall y (Bxy \wedge Mx \wedge \forall z ((Bzy \wedge Mz) \rightarrow z = x) \wedge Px)$

Prem	1.	$\exists x \exists w \forall y (Bxy \wedge Tx \wedge \forall z ((Bzy \wedge Tz) \rightarrow z = x) \wedge Px \wedge Bwy \wedge Mw \wedge \forall z ((Bzy \wedge Mz) \rightarrow z = w) \wedge w = x)$	
Sup	2.	$\forall y (Bay \wedge Ta \wedge \forall z ((Bzy \wedge Tz) \rightarrow z = a) \wedge Pa \wedge Bby \wedge Mb \wedge \forall z ((Bzy \wedge Mz) \rightarrow z = b) \wedge b = a)$	
2	3.	$Bac \wedge Ta \wedge \forall z ((Bzc \wedge Tz) \rightarrow z = a) \wedge Pa \wedge Bbc \wedge Mb \wedge \forall z ((Bzc \wedge Mz) \rightarrow z = b) \wedge b = a$	2, E \forall
2	4.	$Bbc \wedge Mb \wedge \forall z ((Bzc \wedge Mz) \rightarrow z = b)$	3, E \wedge
2	5.	$b = a$	3, E \wedge
2	6.	Pa	3, E \wedge
2	7.	Pb	5, 6, E= $$
2	8.	$Bbc \wedge Mb \wedge \forall z ((Bzc \wedge Mz) \rightarrow z = b) \wedge Pb$	4, 7, I \wedge
2	9.	$\forall y (Bby \wedge Mb \wedge \forall z ((Bzy \wedge Mz) \rightarrow z = b) \wedge Pb)$	8, I \forall
2	10.	$\exists x \forall y (Bxy \wedge Mx \wedge \forall z ((Bzy \wedge Mz) \rightarrow z = x) \wedge Px)$	9, I \exists
1	11.	$\exists x \forall y (Bxy \wedge Mx \wedge \forall z ((Bzy \wedge Mz) \rightarrow z = x) \wedge Px)$	1, 2-10, E \exists

i) $\neg(n = m), \forall x Pmx \vdash \exists x Pxn$

Prem	1.	$\neg(n = m)$	
Prem	2.	$\forall x Pmx$	
2	3.	Pmn	2, E \forall
2	4.	$\exists x Pxn$	3, I \exists

j) $\exists x \exists y (Pxy \wedge \forall z (Pzy \rightarrow z = x) \wedge \neg Ex), \exists x \exists y (Axy \wedge \forall z (Azy \rightarrow z = x) \wedge x = n) \perp$
 $\neg En$

3.

a) $\vdash \forall x (x = x)$ [LEI DA IDENTIDADE]

1.	$a = a$	I=
2.	$\forall x (x = x)$	1, I \forall

b) $\vdash \forall x \forall y ((x = y) \rightarrow (y = x))$ [REFLEXIVIDADE DA IDENTIDADE]

Sup	1.	$a = b$	
	2.	$b = b$	I=
1	3.	$b = a$	1, 2, E=
	4.	$(a = b) \rightarrow (b = a)$	1-3, I \rightarrow
	5.	$\forall x \forall y (x = y \rightarrow (y = x))$	4, I \forall

c) $\vdash \forall x \forall y \forall z (((x = y) \wedge (y = z)) \rightarrow (x = z))$ [TRANSITIVIDADE DA IDENTIDADE]

Sup	1.	$(a = b) \wedge (b = c)$	
1	2.	$a = b$	1, E \wedge
1	3.	$b = c$	1, E \wedge
1	4.	$a = c$	2, 3, E=
	5.	$((a = b) \wedge (b = c)) \rightarrow (a = c)$	1-4, I \rightarrow
	6.	$\forall x \forall y \forall z (((x = y) \wedge (y = z)) \rightarrow (x = z))$	5, I \forall

4.

a) $\forall x \exists y (Px \rightarrow ((x = y) \wedge Ey)), \forall x (Ex \rightarrow Lx) \vdash \forall x (Px \rightarrow Lx)$

Prem	1. $\forall x \exists y (Px \rightarrow ((x = y) \wedge Ey))$	
Prem	2. $\forall x (Ex \rightarrow Lx)$	
1	3. $\exists y (Pa \rightarrow ((a = y) \wedge Ey))$	1, E \forall
Sup	4. $\neg \forall x (Px \rightarrow Lx)$	
Sup	5. $Pa \rightarrow ((a = b) \wedge Eb)$	
4	6. $\exists x \neg (Px \rightarrow Lx)$	4, RD: $\neg \forall$
Sup	7. $\neg (Pa \rightarrow La)$	
7	8. $Pa \wedge \neg La$	7, RD: $\neg \rightarrow$
7	9. Pa	7, E \wedge
5, 7	10. $(a = b) \wedge Eb$	5, 9, E \rightarrow
5, 7	11. $a = b$	10, E \wedge
5, 7	12. Eb	10, E \wedge
5, 7	13. Ea	11, 12, E $=$
2	14. $Ea \rightarrow La$	2, E \forall
2, 5, 7	15. La	13, 14, E \rightarrow
7	16. $\neg La$	8, E \wedge
2, 5, 7	17. $La \wedge \neg La$	15, 16, I \wedge
2, 5, 7	18. $\exists x Lx \wedge \neg \exists x Lx$	17, I \exists
2, 4, 5	19. $\exists x Lx \wedge \neg \exists x Lx$	6, 7-18, E \exists
1, 2, 4	20. $\exists x Lx \wedge \neg \exists x Lx$	3, 5-19, E \exists
1, 2	21. $\forall x (Px \rightarrow Lx)$	4-20, I \neg

b) $Pn, \exists x (Cxn \wedge \forall y (Cyn \rightarrow y = x) \wedge \neg Px) \vdash \exists x (Cxn \wedge \forall y (Cyn \rightarrow y = x) \wedge \neg n = x)$

Prem	1. Pn	
Prem	2. $\exists x (Cxn \wedge \forall y (Cyn \rightarrow y = x) \wedge \neg Px)$	
Sup	3. $Can \wedge \forall y (Cyn \rightarrow y = a) \wedge \neg Pa$	
3	4. $\neg Pa$	3, E \wedge
Sup	5. $n = a$	
1, 5	6. Pa	1, 5, E $=$
1, 3, 5	7. $Pa \wedge \neg Pa$	4, 6, I \wedge
1, 3	8. $\neg n = a$	5-7, I \neg
3	9. $Can \wedge \forall y (Cyn \rightarrow y = a)$	3, E \wedge

1, 3	10. $Can \wedge \forall y (Cyn \rightarrow y = a) \wedge \neg n = a$	8, 9, $I \wedge$
1, 2	11. $\exists x (Cxn \wedge \forall y (Cyn \rightarrow y = x) \wedge \neg n = x)$	2, 3-10, $E \exists$